# 211. The Crystal and Molecular Structure of Certain Dicarboxylic Acids. Part VI. Sebacic Acid. 

By J. D. Morrison and J. Monteath Robertson.


#### Abstract

The crystal structure of sebacic acid has been fully determined by quantitative $X$-ray analysis, and is found to be similar to that of adipic acid. The finer details of the molecular structure are also found to be similar, and there is an indication of certain minor bond-length variations extending over the entire length of the molecule. In the grouping of the molecules in the crystal there is again evidence of a fairly close lateral approach, as well as the usual end-to-end hydrogen bonding through the carboxyl groups. This type of lateral approach appears to be characteristic of the acids of the " even " series. The effect of the hydrogen atoms on the intensities of the $X$-ray reflections is discussed, and it is shown that they make considerable contributions to the structure factors of the large spacing planes. Their presence is also evident on some of the contour maps.


The first three members of the series of normal aliphatic dicarboxylic acids (oxalic, malonic, and succinic) have widely different unit cells and crystal structures. From glutaric acid upwards, however, it has been shown, chiefly by the work of Caspari ( $J$., 1928, 3235; 1929, 2709), that the acids form a well-defined and closely related crystallographic series, composed of two sub-groups, the acids with an even and those with an odd number of carbon atoms in the molecule. The dimensions of the $a$ and $b$ axes and the angle $\beta$ are rather similar for both groups, and almost constant for successive members in each group, the chief variant being the $c$ axis. The even acids have two centrosymmetric molecules in the unit cell, and the $c$ axis gives a measure of the chain length. The odd acids have four molecules in the unit cell, and the length of the $c$ axis in this case corresponds to two molecules lying end to end.

In the previous paper (Part V) we have examined the fine structure of adipic acid which may be described as the first normal member of the even series, and have obtained evidence of certain bond-length variations in the saturated chain. To discover whether these effects persist in the higher members of the even series we have now made a similar quantitative $X$-ray investigation of the structure of sebacic acid. Our measurements lead to the rather surprising conclusion that in this saturated chain of ten carbon atoms there are again variations, in the nature of alternate long and slightly shorter bonds, over the entire length of the molecule. It is true that these variations are not much greater than the possible experimental error, but their regularity and their persistence in other members of this series of even acids would seem to be significant.

Another point of interest in this analysis concerns the effect of the hydrogen atoms on the intensities of the $X$-ray reflections. These are usually neglected in $X$-ray work, but if as a first approximation we take the scattering power of an atom to be proportional to its atomic number, then hydrogen is responsible for over $16 \%$ of the total scattering matter in sebacic acid. At low angles of reflection this effect should be noticeable, and we find that it has in fact a considerable influence on certain reflections, as well as being apparent in the electron-density maps. We make allowance for this effect in our calculations.

Crystal Data.-Sebacic acid, $\mathrm{C}_{10} \mathrm{H}_{18} \mathrm{O}_{4} ; M, 202 \cdot 2$; m. p. $134^{\circ}$; d, calc. $1 \cdot 226$, found $1 \cdot 231$. Monoclinic prismatic, $a=10 \cdot 10 \pm 0.03, b=5.00 \pm 0.01, c=15 \cdot 10 \pm 0.05 \mathrm{~A} ., \beta=133.8^{\circ} \pm 0.3^{\circ}$. Absent spectra, ( $h 0 l$ ) when $h$ is odd, ( $0 k 0$ ) when $k$ is odd. Space group $C_{2 h}^{5}\left(P 2_{1} / a\right)$. Two molecules per unit cell. Molecular symmetry, centre. Volume of the unit cell, 548 A. ${ }^{3}$. Absorption coefficient for $X$-rays $(\lambda=1.54) \mu=9.08 \mathrm{~cm} .^{-1}$. Total number of electrons per unit cell $=\mathrm{F}(000)=220$.

The crystals obtained from most solvents are in the form of very thin flakes developed on the ( 001 ) face. These are generally unsuitable for $X$-ray work. Slow and careful crystallisation from amyl acetate, however, sometimes yields crystalline fragments which can be cut to give suitable specimens. Crystals so obtained usually show only the (001) and (100) faces, no end faces being identified with certainty. There is marked cleavage on (001) and (100). Further goniometric and optical data are given by Caspari (loc. cit.).

Analysis of the Structure.-Probable positions for the molecules in this crystal have been suggested by Caspari (loc. cit.) but the work does not lead to the actual determination of any atomic positions. The asymmetric unit in the structure consists of one half molecule. The dimensions of the $a$ and $b$ axes are almost identical with those of adipic acid (Part V), and it is clear that the molecular arrangement in the two acids is similar, the additional four carbon atoms in the chain of sebacic acid being accommodated by the increased $c$ axis. The similarity

of the $b$ axes in the two structures also indicates that the inclination of the molecules to the (010) plane must be approximately the same.

From a consideration of these facts, and an examination of the planes of outstanding intensity, notably the $(200),(006),(40 \overline{6}),(40 \overline{1} \overline{2}),(20 \overline{6})$, and $(20 \overline{1} \overline{2})$ planes, it was possible to find by trial a structure which gave fair agreement between the calculated and measured structure factors. In this structure the whole molecule was assumed to be planar. The co-ordinates were then refined by successive double Fourier syntheses, giving projections of the structure on the (010) plane. The final projection is shown in Fig. 1, and from it accurate values of the $x$ and $z$ co-ordinates of all the atoms except $\mathrm{C}(5)$ and $\mathrm{O}(1)$ can be obtained.

Fig. 2.


The $y$ co-ordinates were first obtained by trial, and then refined by a further Fourier projection made along the [101] axis using the ( $h k \bar{h}$ ) structure factors. This is shown in Fig. 2. All the atoms are resolved in this case except C(5), the position of which is again uncertain. In this projection the atoms are rather crowded and mutual interference tends to reduce the accuracy. Nevertheless, these two projections together give a fairly precise picture of the complete structure.

Orientation, Co-ordinates, and Dimensions.-The mid points of the seven most central carbon-carbon bonds in the molecule are found to lie very accurately on a straight line, which is accordingly chosen as the principal molecular axis, $L$. The eight central carbon atoms are also very nearly coplanar ( $\pm 0.03 \mathrm{~A}$.), and so the second molecular axis $M$ is chosen in this plane, at right angles to $L$. The third axis $N$ is perpendicular to $L$ and $M$. The orientation of the molecule in the crystal can then be expressed by giving the angles $(\chi, \psi, \omega)$ which these molecular axes make with the crystallographic axes $a, b$, and $c^{\prime}, c^{\prime}$ being chosen perpendicular to $a$ and $b$. These angles are given in Table I.

The co-ordinates of the atoms in the asymmetric crystal unit (half the chemical molecule) are given in Table II, as determined from the final Fourier projections. The figures for atom $\mathbf{C}(5)$ are perhaps more in doubt than the others, owing to lack of satisfactory resolution, but
the position assigned is fairly well defined by the outer contour levels and cannot be much in doubt. These co-ordinates are referred to the crystallographic axes.

Table I.
Orientation of the molecule in the crystal.


Table II.
Co-ordinates. Centre of symmetry as origin.
$x, y, z$ are referred to the monoclinic crystal axes, and $x^{\prime}, y, z^{\prime}$ to the orthogonal axes $a, b$, and $c^{\prime}$.


Table IV.
Molecular dimensions.


Table V.
Intermolecular distances (A.).*
$\mathrm{O}(1 A) \cdots \mathrm{O}(2 C)=2 \cdot 68 \pm 0.05$
$\mathrm{O}(2 A) \cdots \mathrm{O}(C)=2 \cdot 68 \quad, \quad$
$\mathrm{O}(2 C) \cdots \mathrm{C}(4 B)=3 \cdot 32 \quad$,
$\mathrm{O}(2 A) \cdots \mathrm{O}(2 C)=3 \cdot 56 \quad$,
$\mathrm{O}(2 C) \cdots$,

| O |  | $=3.78$ |  |
| :---: | :---: | :---: | :---: |
| $\mathrm{O}(1 A)$ | . $\mathrm{C}(2 B)$ | $=3.87$ |  |
| $\mathrm{O}(1 A)$ | . $\mathrm{C}(4 B)$ | $=3.90$ |  |
| $\mathrm{O}(1 A)$ | . $\mathrm{C}(3 B)$ | $=3.96$ |  |

* $A=$ standard molecule ; $B=$ reflected molecule ; $C=$ molecule one translation along $c$ axis.

For many purposes it is convenient to list the co-ordinates of the atoms with reference to the molecular axes, $L, M$, and $N$, as defined above. These figures are collected in Table III.

The bond lengths and valency angles within the molecule deduced from these co-ordinates are shown in Fig. 3, and collected in Table IV. The central carbon-carbon bond at the centre of symmetry appears to have the normal length of 1.54 A . After that there is a distinct although slight alternation of long and short bonds extending over the entire length of the molecule. The final bonds connecting to the carboxyl groups, $C(4)-C(5)$, are somewhat uncertain but appear to be slightly longer than the bonds $C(3)-C(4)$ immediately adjacent to them. The structure of the carboxyl group is closely similar to that found in the other acids of the series.

The geometry of the molecule is also very similar to that of the other members of the series, except that in sebacic acid the whole molecule appears to be more strictly planar. The eight
central carbon atoms, $\mathrm{C}(1), \mathrm{C}(2), \mathrm{C}(3), \mathrm{C}(4), \mathrm{C}\left(1^{\prime}\right), \mathrm{C}\left(2^{\prime}\right), \mathrm{C}\left(3^{\prime}\right)$, and $\mathrm{C}\left(4^{\prime}\right)$, probably lie accurately in one plane ( $\pm 0.03 \mathrm{~A}$.). The plane of the carboxyl group is tilted by approximately $3^{\circ}$ from this plane, and the carboxyl-group plane then passes through the centre of symmetry between this molecule and the next molecule along the $c$ axis and contains all the atoms of the next adjacent carboxyl group.

Intermolecular Distances.-The general pattern of the intermolecular approach distances, which is indicated by the line drawings in Figs. 1 and 2, is remarkably similar to that found in adipic acid (Part V). In particular, the hydrogen bridges between adjacent carboxyl groups are of the same length ( 2.68 A .). We also find evidence of an unusually short approach distance ( 3.32 A .) between the carboxyl oxygen atom $\mathrm{O}(2)$ and the carbon atom $\mathrm{C}(4)$ of the reflected molecule one translation further along the $c$ axis. In adipic acid this distance was 3.29 A . The other approach distances are greater than 3.6 A . The values are collected in Table V.

Effect of Hydrogen Atoms on the Intensities of the X-Ray Reflec-tions.-In computing the structure factors for this crystal from the final co-ordinates (see Table VIII), it was noted that the agreement with the measured values for certain large spacing planes remained distinctly poor in spite of careful corrections to the intensities and refinement of the atomic co-ordinates. Further investigation showed that most of these discrepancies could be explained by allowing for the hydrogen atoms, which in sebacic acid account for over $16 \%$ of the sum of the atomic numbers.

It is to be expected that hydrogen will make a significant contribution to the total coherent scattering for the low-angle reflections. Atomic scattering curves can be computed, but the allowance for the temperature factor is rather uncertain. In applying the corrections we have accordingly used the following provisional and empirical figures for the hydrogen $f$ curve :

| $\sin \theta(\lambda=1.54)$ |  | 0.0 | $0 \cdot 1$ | 0.2 | 3 | $0 \cdot 4$ | 0.5 | $0 \cdot 6$ | 0.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 100 | 83 | 58 | 36 | 20 | 8.7 | 2.9 |  |

For values of $\sin \theta$ greater than about 0.4 the corrections are not significant. On the basis of this curve each hydrogen contribution

Fig. 3.


Dimensions of the sebacic acid molecule. was given a weight of 1 as against a weight of 6 for each carbon and 9 for each oxygen contribution on the empirical carbon-oxygen curve given in the Experimental section.

Co-ordinates were assigned to the hydrogen atoms attached to the carbon chain by assuming a carbon-hydrogen distance of 1.09 A . and a tetrahedral distribution of the valency bonds. The resulting co-ordinates are shown in Table VI.

Table VI.
Co-ordinates of hydrogen atoms referred to the monoclinic crystal axes and centre of symmetry as origin.

| Atom <br> (cf. Fig. 4). | $\frac{2 \pi x}{a}$. | $\frac{2 \pi y}{b}$. | $\frac{2 \pi z}{c}$. | Atom. <br> (cf. Fig. 4). | $\frac{2 \pi x}{a}$. | $\frac{2 \pi y}{b}$. | $\frac{2 \pi z}{c}$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| H(1) | $-59^{\circ}$ | $30^{\circ}$ | $-9^{\circ}$ | H(5) | $-50^{\circ}$ | $3^{\circ}$ | $55^{\circ}$ |
| H(2) | 14 | 88 | 26 | H(6) | 22 | 61 | 89 |
| H(3) | -6 | -112 | 38 | H(7) | 1 | -140 | 101 |
| H(4) | 66 | -55 | 72 | H(8) | 75 | -82. | 136 |

The effect of making corrections for hydrogen on this basis is shown in Table VII, which lists the structure factors for planes with $\sin \theta$ less than $0 \cdot 4$, and gives the result of the calculation with and without the hydrogen contribution. When the discrepancy is expressed in the usual way as

$$
\frac{\Sigma(\mid F \text { meas. }|-| F \text { calc. } \mid)}{\Sigma \mid F \text { meas } . \mid} \times 100
$$

we obtain a figure of $13.8 \%$ for the uncorrected $F$ values and only $8.4 \%$ for the hydrogen corrected $3 \times$
values. The latter figure is very much better than that usually obtained in $X$-ray investigations of organic structures and seems to indicate that the hydrogen correction is generally significant. It will, of course, be especially important in the case of aliphatic compounds where the proportion of hydrogen in the molecule is relatively high.

TAble VII.
Structure factors for planes with $\sin \theta<0.4(\lambda=1.54$ A.) showing effect of hydrogen.

| $h k l$. | F, meas. | F , calc. with H . | F , calc. without H . | $h k l$. | F, meas. | F, calc. with H . | F, calc. without H . |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 200 | $94 \cdot 0$ | +103.0 | $+99.5$ | 403 | $10 \cdot 0$ | + 11.5 | +11.5 |
| 400 | $15 \cdot 0$ | + 19.0 | +18.0 | 404 | $25 \cdot 0$ | - 24.0 | $-25 \cdot 0$ |
| 001 | $33 \cdot 5$ | - 34.5 | $-47.5$ | $40 \overline{5}$ | $32 \cdot 5$ | + 32.0 | +33.5 |
| 002 | $11 \cdot 0$ | + 12.5 | $+17.0$ | $40 \overline{6}$ | $59 \cdot 0$ | - 54.5 | $-48.0$ |
| 003 | $4 \cdot 0$ | + 4.5 | + 4.5 | 407 | $10 \cdot 0$ | + 8.5 | +11.0 |
| 004 | $9 \cdot 5$ | - 9.5 | $-9.5$ | 011 | 28.5 | - 30.0 | $-26.5$ |
| 005 | $8 \cdot 0$ | + 7.0 | + 7.0 | 012 | $3 \cdot 0$ | + 3.5 | + 2.0 |
| 020 | $2 \cdot 0$ | + 1.5 | + 1.5 | 013 | $2 \cdot 5$ | - 3.0 | - 2.0 |
| 203 | $9 \cdot 0$ | + 9.0 | + 8.5 | 014 | 6.5 | + 6.5 | + 6.0 |
| 202 | $2 \cdot 5$ | - 2.0 | - 2.0 | 015 | $7 \cdot 0$ | + 3.0 | + 1.5 |
| 201 | 18.5 | $-17.5$ | $-17.5$ | 021 | $27 \cdot 0$ | + 26.5 | $+26.0$ |
| 201 | 11.5 | + 11.5 | + 4.0 | 022 | 14.5 | - 11.5 | $-11.5$ |
| 202 | $3 \cdot 0$ | - 4.0 | $0 \cdot 0$ | 023 | 11.5 | + 10.5 | $+11.5$ |
| 203 | 13.5 | + 14.0 | $+15.5$ | 11 I | $8 \cdot 5$ | + 8.0 | + $5 \cdot 0$ |
| 204 | $23 \cdot 5$ | - 24.5 | $-28.5$ | 12I | 11.0 | - 8.5 | $-12.5$ |
| $20 \overline{5}$ | 25.5 | $+22.5$ | $+26.0$ | $21 \overline{2}$ | $15 \cdot 5$ | + 14.5 | $+15 \cdot 0$ |
| 206 | 19.5 | - 19.5 | $-19.0$ | $22 \overline{2}$ | $7 \cdot 0$ | + $5 \cdot 0$ | + 7.0 |
| $20 \overline{7}$ | $<0.5$ | + 0.5 | $0 \cdot 0$ | 313 | $<1.0$ | 0.0 | + 0.5 |
| 401 | $26 \cdot 0$ | + 28.0 | $+29.0$ | $32 \overline{3}$ | $14 \cdot 0$ | + 15.0 | +14.0 |
| 402 | $9 \cdot 0$ | - 8.0 | - 8.0 | 414 | $8 \cdot 0$ | $8 \cdot 0$ | $-7.5$ |

The effect of the hydrogen atoms on the electron-density map is fairly clear in Fig. 1 (see also Fig. 4). After this effect was realised, the co-ordinates assigned to the carbon atoms were varied to see whether the positions assigned might be perturbed by the hydrogens, but within the limits of experimental error this was not found to be the case.

The effect of the hydrogen contribution on planes of large spacing has been noticed before, by Bunn (Trans. Faraday Soc., 1939, 35, 482) and by Hughes and Lipscomb (J. Amer. Chem. Soc., 1946, 68, 1970), but this would seem to be the first case in which it has been shown to have a really extensive effect.

## Experimental.

X-Ray Measurements.-As in the previous investigations in this series, copper $K$-a radiation ( $\lambda=1.54 \mathrm{~A}$.) was used, and the moving-film method was employed to record the intensities.

For the ( $h 0 l$ ) zone the principal crystal specimen, cut from a larger fragment, had cross-section on ( 010 ) of $0.30 \mathrm{~mm} . \times 0.11 \mathrm{~mm}$. Relative absorption corrections were not made, as they were found not to be significant for a specimen of this size completely immersed in the $X$-ray beam. To extend the range of intensities, a larger specimen of $1.20 \mathrm{~mm} . \times 0.62 \mathrm{~mm}$. cross-section was used. Combining the results from these specimens, a total of 141 out of a possible 145 reflections were recorded.

For the ( $k k h$ ) zone, a specimen of cross-section $0.55 \mathrm{~mm} . \times 0.45 \mathrm{~mm}$. normal to the rotation axis was used, and for the ( $0 k l$ ) zone one of $0.28 \mathrm{~mm} . \times 0.25 \mathrm{~mm}$.

The multiple-film technique (Robertson, $J$. Sci. Instr., 1943, 20, 175) was used to correlate the intensities, chiefly by visual estimation, over a total range of about $\mathbf{2 5 , 0 0 0}$ to 1 . In the different zones, the sets of intensities were correlated by means of their common axial reflections. The usual correction factors were applied, and the structure factors are listed in Table VIII. The absolute scale was obtained by correlation with the values finally calculated from the atomic positions.

Fourier Analysis and Structure Factor Calculations.-The electron-density maps were computed by the usual methods (idem, ibid., 1948, 25, 28). For the projection on ( 010 ) the axial subdivisions were $a / 60=0.168$ A., and $c / 60=0.252 \mathrm{~A}$. For the normal projection along the [101] axis the subdivisions were $b / 30=0.167 \mathrm{~A}$. and $a \cos 2 \cdot 1^{\circ} / 60=0.168 \mathrm{~A}$.

A plot of the asymmetric crystal unit in the (010) projection is shown in Fig. 4, with the centres finally assigned to the atoms shown in relation to the co-ordinate network. In this diagram the positions of the centres assumed for the hydrogen atoms are also shown.

From the final co-ordinates listed in Table II all the structure factors were recalculated and the results are given in Table VIII. The results listed there do not take account of the hydrogen contributions, which are dealt with separately for the large spacing planes in Table VII. For the general calculation of the structure factors the following composite empirical $f$-curve, which appeared to give the best agreement with the observed values, was derived :

| $\sin \theta(\lambda=1.54)$ | $0 \cdot 0$ | $0 \cdot 1$ | $0 \cdot 2$ | $0 \cdot 3$ | $0 \cdot 4$ | 0.5 | $0 \cdot 6$ | 0.7 | $0 \cdot 8$ | $0 \cdot 9$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{(0)}$ | 100 | 85 | 62 | 44 | 33 | 25 | 18 | 13 | $8 \cdot 8$ | $5 \cdot 5$ |

On the basis of this composite curve the contributions of the oxygen atoms were weighted relatively to the carbon contributions in the ratio of 1.5 to 1.0 .

The discrepancies finally obtained, expressed as percentages in the usual way, were $16.7 \%$ for the ( $h 0 l$ ) reflections, $13 \cdot 2 \%$ for the ( $h k \bar{h}$ ) reflection, and $15 \cdot 7 \%$ over all. As has been noted above, these agreements can be very greatly improved for the large spacing planes by taking account of the hydrogen contributions.

Fig. 4.


Asymmetric crystal unit. Centres of atoms marked with crosses.
Table VIII.
Measured and calculated values of the structure factor.

|  | $\sin \theta$ |  | $\sin \theta$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| hkl. | ( $\lambda=1.54 \mathrm{~A}$.). | F, meas. | F, calc. | $h k l$. | ( $\lambda=1.54 \mathrm{~A}$ ) | F, meas. | F, calc. |
| 200 | $0 \cdot 212$ | $94 \cdot 0$ | +99.5 | 20 I | $0 \cdot 171$ | 11.5 | + 4.0 |
| 400 | $0 \cdot 423$ | $15 \cdot 5$ | +18.0 | $20 \overline{2}$ | $0 \cdot 154$ | $3 \cdot 0$ | $-0.0$ |
| 600 | $0 \cdot 635$ | 1.0 | $-1.5$ | 203 | $0 \cdot 168$ | $13 \cdot 5$ | +15.5 |
| 800 | $0 \cdot 846$ | 1.5 | $-1.0$ | 204 | $0 \cdot 206$ | $23 \cdot 5$ | $-28.5$ |
|  |  |  |  | 205 | 0.259 | $25 \cdot 5$ | $+26.0$ |
| 001 | $0 \cdot 070$ | 33.5 | -47.5 | 206 | $0 \cdot 319$ | 19.5 | $-19.0$ |
| 002 | $0 \cdot 141$ | 11.0 | +17.0 | 207 | $0 \cdot 383$ | $<0.5$ | $0 \cdot 0$ |
| 003 | $0 \cdot 212$ | $4 \cdot 0$ | + 4.5 | 208 | $0 \cdot 448$ | $9 \cdot 5$ | +8.0 |
| 004 | $0 \cdot 282$ | $9 \cdot 5$ | $-9.5$ | 209 | 0.514 | $10 \cdot 5$ | $-9 \cdot 0$ |
| 005 | $0 \cdot 353$ | $8 \cdot 0$ | + $7 \cdot 0$ | 2,0,10 | $0 \cdot 584$ | $7 \cdot 0$ | + 4.5 |
| 006 | $0 \cdot 423$ | 18.5 | +16.5 | 2,0, 1 I | $0 \cdot 653$ | $8 \cdot 0$ | - 7.5 |
| 007 | $0 \cdot 494$ | $15 \cdot 0$ | $-14.0$ | 2,0,12 | 0.720 | 21.5 | $-19.5$ |
| 008 | $0 \cdot 564$ | $13 \cdot 5$ | +14.0 | 2,0,13 | 0.789 | $9 \cdot 5$ | + 8.5 |
| 009 | $0 \cdot 635$ | $8 \cdot 5$ | $-8.5$ | 2,0,14 | $0 \cdot 858$ | $4 \cdot 5$ | $-4.0$ |
| 0,0,10 | $0 \cdot 705$ | $5 \cdot 0$ | + $5 \cdot 0$ | 2,0,15 | 0.928 | 1.5 | + 1.0 |
| 0,0,11 | $0 \cdot 776$ | $10 \cdot 0$ | $-9.5$ |  |  |  |  |
| 0,0,12 | $0 \cdot 846$ | $5 \cdot 0$ | $-5.5$ | 409 | 0.978 | 1.5 | +1.5 |
| 0,0,13 | 0.917 | $2 \cdot 0$ | + 1.5 | 408 | 0.910 | 1.0 | $-0.5$ |
| 0,0,14 | 0.987 | 0.5 | $0 \cdot 0$ | 407 | 0:845 | 1.0 | $-1.0$ |
|  |  |  |  | 406 | 0.780 | $2 \cdot 0$ | + 4.0 |
| 020 | $0 \cdot 308$ | $2 \cdot 0$ | +1.5 | 405 | 0.715 | $10 \cdot 0$ | +11.0 |
| 040 | $0 \cdot 616$ | <2.5 | - $2 \cdot 0$ | 404 | $0 \cdot 653$ | $4 \cdot 0$ | - 4.0 |
| 060 | 0.924 | $2 \cdot 0$ | $-3 \cdot 0$ | 403 | 0.589 | $6 \cdot 0$ | + 8.5 |
|  |  |  |  | 402 | $0 \cdot 530$ | $9 \cdot 5$ | $-11.0$ |
| 2,0,11 | 0.938 | $4 \cdot 0$ | $-4.5$ | 401 | $0 \cdot 478$ | $5 \cdot 0$ | + 6.5 |
| 2,0,10 | $0 \cdot 868$ | 1.0 | - 0.0 | 401 | $0 \cdot 380$ | 26.0 | +29.0 |
| 209 | 0.798 | $<1.0$ | $-1.5$ | 402 | $0 \cdot 344$ | $9 \cdot 0$ | -8.0 |
| 208 | 0.728 | $4 \cdot 0$ | +6.0 | 403 | $0 \cdot 319$ | $10 \cdot 0$ | +11.5 |
| 207 | $0 \cdot 659$ | $8 \cdot 0$ | $-10.0$ | 404 | $0 \cdot 308$ | $25 \cdot 0$ | $-25.0$ |
| 206 | $0 \cdot 588$ | $10 \cdot 5$ | $+15.5$ | $40 \overline{5}$ | $0 \cdot 314$ | $32 \cdot 5$ | +33.5 |
| 205 | $0 \cdot 521$ | $8 \cdot 5$ | +8.5 | 406 | $0 \cdot 335$ | $59 \cdot 0$ | $-48.0$ |
| 204 | $0 \cdot 455$ | $7 \cdot 5$ | $-6.5$ | 407 | $0 \cdot 369$ | $10 \cdot 0$ | +11.0 |
| 203 | $0 \cdot 388$ | $9 \cdot 0$ | + 8.5 | 408 | $0 \cdot 410$ | 1.0 | + 0.5 |
| 202 | $0 \cdot 323$ | $2 \cdot 5$ | - 2.0 | 409 | $0 \cdot 463$ | $7 \cdot 5$ | $-7 \cdot 0$ |
| 201 | $0 \cdot 263$ | 18.5 | $-17.5$ | 4,0,10 | 0.518 | $7 \cdot 5$ | $+5.5$ |

Table VIII.-continued.

| $\sin \theta$ |  |  | $\sin \theta$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $h k l$. | ( $\lambda=1.54$ ). | $F$, meas. | F, calc. | $h k l$. | ( $\lambda=1 \cdot 54$ ) | F , meas. | F, calc. |
| 4,0,II | 0.576 | $6 \cdot 0$ | - 4.5 | 10,0, $1 \overline{7}$ | 0.902 | $1 \cdot 0$ | $+1.0$ |
| 4,0,12 | $0 \cdot 638$ | 36.5 | $-29.0$ | 10,0,18 | 0.940 | $8 \cdot 0$ | + 4.0 |
| 4,0, $\overline{1} 3$ | $0 \cdot 698$ | $11 \cdot 0$ | $+10.5$ | 10,0,19 | 0.983 | $3 \cdot 0$ | $+0.5$ |
| 4,0,14 | 0.761 | $8 \cdot 0$ | - 6.0 |  |  |  |  |
| 4,0, 15 | 0.825 | $3 \cdot 0$ | + 2.5 | 12,0,7 | 0.990 | $2 \cdot 5$ | $-2.5$ |
| 4,0, $\overline{1} \overline{6}$ | $0 \cdot 893$ | $<1.0$ | $0 \cdot 0$ | 12,0,8 | 0.978 | $2 \cdot 5$ | $-3.5$ |
| 4,0,17 | $0 \cdot 960$ | $3 \cdot 0$ | $-1.0$ | 12,0,9 | 0.958 | $4 \cdot 0$ | + 4.0 |
|  |  |  |  | 12,0,10 | 0.940 | $3 \cdot 0$ | - 2.5 |
| 606 | 0.979 | $0 \cdot 5$ | $+0.5$ | 12,0,II | 0.930 | 1.5 | $+1.5$ |
| 605 | 0.918 | $4 \cdot 0$ | + 5.0 | 12,0,12 | 0.925 | 1.0 | $-0.5$ |
| 604 | $0 \cdot 858$ | $2 \cdot 5$ | + 2.0 | 12,0,13 | 0.925 | $2 \cdot 0$ | $-1.5$ |
| 603 | 0.795 | $2 \cdot 5$ | +1.0 | 12,0,14 | 0.930 | 1.0 | $-1.0$ |
| 602 | 0.740 | $2 \cdot 0$ | $-5.0$ | 12,0,15 | 0.941 | $2 \cdot 0$ | $+2.0$ |
| 601 | $0 \cdot 688$ | $3 \cdot 5$ | + 6.0 | 12,0,16 | 0.958 | $2 \cdot 5$ | - 2.0 |
| 601 | $0 \cdot 590$ | 14.5 | $+17 \cdot 0$ | 12,0,17 | 0.978 | $2 \cdot 5$ | $+2.0$ |
| 602 | $0 \cdot 549$ | 1.0 | +1.5 |  |  |  |  |
| $60 \overline{3}$ | 0.514 | $4 \cdot 0$ | - 4.0 | 011 | $0 \cdot 170$ | 28.5 | -26.5 |
| 604 | $0 \cdot 488$ | $2 \cdot 0$ | - 2.5 | 012 | $0 \cdot 209$ | $3 \cdot 0$ | + $2 \cdot 0$ |
| 605 | $0 \cdot 470$ | 11.0 | +11.5 | 013 | $0 \cdot 262$ | $2 \cdot 5$ | $-2.0$ |
| 606 | $0 \cdot 463$ | 29.5 | $-30.0$ | 014 | $0 \cdot 321$ | $6 \cdot 5$ | +6.0 |
| $60 \overline{7}$ | $0 \cdot 465$ | $6 \cdot 0$ | $-6.0$ | 015 | $0 \cdot 384$ | $7 \cdot 0$ | +1.5 |
| $60 \overline{8}$ | $0 \cdot 479$ | $5 \cdot 0$ | + 5.0 | 016 | $0 \cdot 450$ | $16 \cdot 5$ | $+21.0$ |
| 609 | $0 \cdot 503$ | $10 \cdot 0$ | $-10.0$ | 017 | 0.516 | $7 \cdot 5$ | $-7.5$ |
| 6,0,10 | $0 \cdot 535$ | 11.5 | $+11.0$ | 018 | $0 \cdot 585$ | $<2 \cdot 5$ | + $2 \cdot 0$ |
| 6,0,11 | 0.573 | $10 \cdot 0$ | $-9.0$ | 019 | $0 \cdot 654$ | $<2 \cdot 5$ | $+1.5$ |
| 6,0,12 | $0 \cdot 668$ | 22.5 | $-19.0$ | 0,1,10 | 0.721 | $3 \cdot 0$ | - 2.0 |
| 6,0,13 | $0 \cdot 686$ | 3.5 | $-1.0$ | 0,1,11 | 0.788 | $3 \cdot 0$ | $-3.0$ |
| 6,0,14 | $0 \cdot 719$ | 1.5 | $-1.0$ | 0,1,12 | $0 \cdot 861$ | $6 \cdot 5$ | + $7 \cdot 0$ |
| 6,0,15 | 0.773 | 1.0 | + 0.5 | 0,1,13 | $0 \cdot 928$ | $3 \cdot 0$ | $-2.5$ |
| 6,0,1 $\overline{6}$ | 0.831 | $2 \cdot 5$ | + 2.0 | 021 | $0 \cdot 315$ | $27 \cdot 0$ |  |
| 6,0, 17 | 0.891 0.953 | 4.5 7.0 | + -1.5 $+\quad 4.0$ | 022 | 0.315 0.339 | 14.5 | +26.0 -11.5 |
| 6;0,18 | 0.953 | $7 \cdot 0$ | + 4.0 | 023 | $0 \cdot 373$ | 11.5 | $+11.5$ |
| 803 | 0.990 | $2 \cdot 5$ | $-1.5$ | 024 | 0.417 0.468 | 9.0 28.0 | +7.5 +28.0 |
| 802 | $0 \cdot 950$ | $<0.5$ | + 0.5 | 025 | 0.468 0.523 | $28 \cdot 0$ 10.0 | +28.0 -13.0 |
| 801 | $0 \cdot 900$ | 1.0 | +1.0 | 027 | 0.581 | 10.0 $<2.5$ | -13.0 $-\quad 2.0$ |
| 801 | $0 \cdot 803$ | $4 \cdot 5$ | + 4.0 | 028 | $0 \cdot 642$ | $<2.5$ | - 0.5 |
| 802 | 0.758 0.720 | $2 \cdot 0$ $5 \cdot 5$ | a +5.5 -8.5 | 029 | 0.710 | <3.0 | a +2.5 +3.5 |
| 804 | $0 \cdot 688$ | $6 \cdot 0$ | + $7 \cdot 0$ | $0,2,10$ $0,2,11$ | 0.770 0.834 | 3.0 $<2.5$ | + 3.5 +1.5 |
| $80 \overline{5}$ | $0 \cdot 660$ | $3 \cdot 5$ | - 3.5 | $0,2,11$ $0,2,12$ | 0.834 0.900 | $<2.5$ 5.0 | $\begin{array}{r}\text { a } \\ +15 \\ \hline 4.5\end{array}$ |
| $80 \overline{6}$ 80 | 0.636 0.623 | 3.5 13.5 | - 6.0 | 0,2,13 | 0.966 | -1.5 | - 0.0 |
| 807 | $0 \cdot 623$ 0.618 | 13.5 5.0 | -16.5 +4.5 | 031 | $0 \cdot 467$ | $3 \cdot 5$ | + 4.0 |
| 809 | $0 \cdot 618$ | $5 \cdot 5$ | - 6.0 | 032 | 0.483 | 11.5 | $\pm 12.5$ |
| 8,0,10 | $0 \cdot 628$ | $7 \cdot 5$ | + 9.0 | 033 | $0 \cdot 507$ | $5 \cdot 0$ | + $5 \cdot 0$ |
| 8,0,11 | $0 \cdot 645$ | $9 \cdot 0$ | $-9.5$ | 034 | 0.541 | $10 \cdot 0$ | +11.0 |
| 8,0,12 | $0 \cdot 670$ | $3 \cdot 0$ | $-3.5$ | 035 | 0.581 | $2 \cdot 5$ | +3.0 |
| $8,0,1 \overline{3}$ | 0.700 | 11.5 | $-10.5$ | 036 | $0 \cdot 626$ | $3 \cdot 0$ | + 4.0 |
| 8,0,14 | 0.738 | 3.5 | $\begin{array}{r}\text { a } \\ +2.5 \\ \hline\end{array}$ | 037 | 0.676 | $8 \cdot 0$ | +8.5 |
| 8,0,15 | 0.777 0.823 | 1.0 | - 0.0 | 038 | 0.729 | $4 \cdot 5$ | $-3.5$ |
| $8,0,16$ | $0 \cdot 823$ | 1.5 4.0 | + 1.5 | 039 | 0.785 | $5 \cdot 0$ | + $5 \cdot 5$ |
| $8,0,17$ | 0.873 0.922 | $4 \cdot 0$ 12.5 | $+\quad 2.5$ $+\quad 7.0$ | 0,3,10 | $0 \cdot 843$ | <2.5 | -4.0 |
| 8,0, 18 | 0.922 | 12.5 | + $7 \cdot 0$ | 0,3,11 | 0.903 | <2.0 | + 2.5 |
| 8,0,19 | 0.978 | $4 \cdot 5$ | $-2.0$ | 0,3,12 | 0.964 | $<1.5$ | $+0.5$ |
| 10,0,2 | 0.970 | $0 \cdot 5$ | $+1.0$ | 041 | $0 \cdot 619$ | $<2.5$ | -1.0 |
| 10,0,3 | 0.928 | $1 \cdot 0$ | - 2.5 | 042 | $0 \cdot 632$ | 2.5 | + 2.5 |
| 10,0,4 | $0 \cdot 889$ | $2 \cdot 5$ | + 4.0 | 043 | $0 \cdot 651$ | $6 \cdot 0$ | $-7.5$ |
| 10,0,5 | $0 \cdot 858$ | $2 \cdot 5$ | -4.0 | 044 | $0 \cdot 677$ | 13.5 | $+18.0$ |
| 10,0,6 | $0 \cdot 828$ | 1.5 | $+0.5$ | 045 | $0 \cdot 709$ | $5 \cdot 5$ | $-6.0$ |
| 10,0,7 | $0 \cdot 806$ | $9 \cdot 5$ | $-10.5$ | 046 | 0.747 | $4 \cdot 5$ | $+3 \cdot 5$ |
| 10,0,8 | $0 \cdot 788$ | $3 \cdot 0$ | $-2.5$ | 047 | 0.789 | $5 \cdot 0$ | $-5 \cdot 5$ |
| 10,0,9 | $0 \cdot 776$ | $4 \cdot 5$ | + $3 \cdot 0$ | 048 | 0.835 | $3 \cdot 5$ | - $3 \cdot 5$ |
| 10,0,10 | $0 \cdot 770$ | $2 \cdot 0$ | $+0.5$ | 049 | $0 \cdot 885$ | $3 \cdot 5$ | $+2 \cdot 0$ |
| 10,0,11 | $0 \cdot 771$ | 1.5 | $-2.5$ | 0,4,10 | 0.936 | $<2 \cdot 0$ | + 1.5 |
| 10,0,1] | $0 \cdot 778$ | 1.0 | + 0.5 |  |  |  |  |
| 10,0,13 | 0.793 | $8 \cdot 0$ | - 7.5 | 051 | 0.773 | <2.5 | $+1.5$ |
| 10,0,14 | $0 \cdot 811$ | 1.0 | + 0.5 | 052 | 0.782 | $<2 \cdot 5$ | $+1.5$ |
| 10,0,15 | 0.835 | $2 \cdot 5$ | + 2.5 | 053 | 0.798 | $3 \cdot 0$ | + $4 \cdot 0$ |
| 10,0,16 | $0 \cdot 866$ | 3-0 | $-4.5$ | 054 | $0 \cdot 820$ | $2 \cdot 5$ | + 2.5 |

Table VIII.-continued.

| $\sin \theta$ |  |  | $\sin \theta$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $h k l$. | ( $\lambda=1.54$ ). | $F$, meas. | F, calc. | $h k l$. | ( $\lambda=1 \cdot 54$ ). | F, meas. | F, calc. |
| 055 | 0.846 | $<2 \cdot 5$ | - 2.0 | 515 | $0 \cdot 413$ | $10 \cdot 0$ | + 9.0 |
| 056 | 0.878 | $<2 \cdot 5$ | $+1.5$ | $52 \overline{5}$ | $0 \cdot 490$ | 10.5 | $+11.0$ |
| 057 | 0.914 | $<2.0$ | $-0.5$ | 535 | $0 \cdot 600$ | $4 \cdot 0$ | $+6.5$ |
| 058 | 0.953 | $2 \cdot 0$ | + 3.0 | 545 | 0.725 | $8 \cdot 0$ | + 9.0 |
|  |  |  |  | 555 | $0 \cdot 860$ | $<1.5$ | $-1.0$ |
| 061 | 0.926 | $2 \cdot 0$ | + 2.5 | $56 \overline{5}$ | 0.999 | $<0.5$ | $-1.5$ |
| 062 | 0.934 | $2 \cdot 5$ | - 3.0 |  |  |  |  |
| 063 | 0.947 | $<1.5$ | -0.5 | ${ }_{626}^{61}$ | 0.484 0.553 | 9.5 $<1.5$ | -11.5 $+\quad 1.5$ |
| 064 | 0.965 | $<1 \cdot 5$ | + 0.5 +0.0 | ${ }_{636}^{6}$ | 0.650 | $<1.5$ 2.0 | +1.5 +4.5 |
| 065 | 0.989 | $<1.0$ | $-0.0$ | $64 \overline{6}$ | 0.768 | $<1.5$ | +1.5 +1.5 |
| 111 | $0 \cdot 189$ | $8 \cdot 5$ | $+5 \cdot 0$ | 656 | $0 \cdot 895$ | $2 \cdot 0$ | $-2.0$ |
| 12I | $0 \cdot 326$ | 11.0 | $-12.5$ | 717 | 0.557 | $9 \cdot 0$ | $-8.5$ |
| 131 | $0 \cdot 474$ | $4 \cdot 5$ | + 4.0 | 727 | $0 \cdot 618$ | $5 \cdot 5$ | + 6.5 |
| 14 I | $0 \cdot 625$ | $2 \cdot 5$ | + 3.5 | 737 | $0 \cdot 708$ | $2 \cdot 5$ | - 4.0 |
| 15I | $0 \cdot 776$ | $5 \cdot 0$ | + 5.0 | 747 | 0.815 | $<1.5$ | $-1.0$ |
| 16 I | 0.930 | $<1.0$ | $0 \cdot 0$ | $75 \overline{7}$ | 0.936 | $<1.0$ | $-1.0$ |
| $21 \overline{2}$ | $0 \cdot 218$ | $15 \cdot 5$ | +15.0 | 818 | $0 \cdot 632$ | 6.5 | + 5.5 |
| $22 \overline{2}$ | $0 \cdot 344$ | $7 \cdot 0$ | + 7.0 | 828 | $0 \cdot 685$ | 12.5 | $-12.5$ |
| 232 | $0 \cdot 486$ | $14 \cdot 5$ | $-14.5$ | 838 | 0.767 | 7.5 | +9.0 |
| 242 | $0 \cdot 635$ | $5 \cdot 5$ | $-6.0$ | 848 | $0 \cdot 867$ | $<1.5$ | +1.0 |
| $25 \overline{2}$ | 0.784 | $4 \cdot 0$ | + 4.5 | 858 | 0.984 | $<0.5$ | $-0.5$ |
| $26 \overline{2}$ | 0.935 | 2.5 | $-2.5$ | $91 \overline{9}$ | $0 \cdot 705$ | 4.5 | + 3.5 |
| 313 | $0 \cdot 277$ | <1.0 | $+0.5$ | 929 | 0.755 | $7 \cdot 0$ | -6.0 |
| $32 \overline{3}$ | $0 \cdot 384$ | 14.0 | + 14.0 | 939 | 0.830 | $3 \cdot 0$ | $-1.5$ |
| $33 \overline{3}$ | $0 \cdot 515$ | $4 \cdot 0$ | -2.5 | 949 | 0.924 | $<1 \cdot 0$ | 0.5 |
| $34 \overline{3}$ | $0 \cdot 657$ | $2 \cdot 0$ | + 2.5 | 10,1,10 | 0.780 | $5 \cdot 0$ | +6.0 |
| $35 \overline{3}$ | $0 \cdot 807$ | $3 \cdot 5$ | -3.5 | 10,2,10 | $0 \cdot 824$ | <1.5 | $0 \cdot 0$ |
| $36 \overline{3}$ | $0 \cdot 950$ | $3 \cdot 5$ | + 3.5 | 10,3,10 | 0.892 | $<1.0$ | $+1.5$ |
|  |  |  |  | 10,4,10 | 0.982 | $<0.5$ | + 0.5 |
| 414 | $0 \cdot 343$ | $8 \cdot 0$ | $-7.5$ | 11,1,1] | $0 \cdot 855$ | $<1.5$ | $-1.0$ |
| 424 | $0 \cdot 434$ | $9 \cdot 0$ | +8.0 | 11,2,11 | $0 \cdot 896$ | $<1.0$ | $-2.0$ |
| $43 \overline{4}$ | $0 \cdot 559$ | 6.5 | $-7.0$ | 11,3,Ī | 0.960 | $<1.0$ | $-1.5$ |
| $44 \overline{4}$ | $0 \cdot 687$ | $6 \cdot 5$ | + 5.0 |  |  |  |  |
| $45 \overline{4}$ | 0.827 | $3 \cdot 5$ | + 4.5 | 12,1,12 | 0.930 | $<1 \cdot 0$ | $-0.5$ |
| $46 \overline{4}$ | 0.973 | $2 \cdot 0$ | + 2.5 | 12,2,12 | $0 \cdot 969$ | $<1 \cdot 0$ | $-0.5$ |

We thank Imperial Chemical Industries Ltd. for a supply of pure sebacic acid, and for a grant which enabled one of us (J. D. M.) to take part in this work.

